EXERCISE – III | SUBJECTIVE QUESTIONS

1. Construct a 3 × 2 matrix whose elements are given by $a_{ij} = 2i - j$.

2. If $\begin{bmatrix} x-y & 1 & z \\ 2x-y & 0 & w \end{bmatrix} = \begin{bmatrix} -1 & 1 & 4 \\ 0 & 0 & 5 \end{bmatrix}$, find x, y, z, w.

3. If $A = \begin{bmatrix} 1 & 2 \\ 3 & -4 \\ 5 & 6 \end{bmatrix}$ and $B = \begin{bmatrix} 4 & 5 & 6 \\ 7 & -8 & 2 \end{bmatrix}$, will AB be equal to BA. Also find AB & BA.

4. If $A = \begin{bmatrix} 3 & -4 \\ 1 & -1 \end{bmatrix}$ show that $A^k = \begin{bmatrix} 1+2k & -4k \\ k & 1-2k \end{bmatrix}$

5. (i) Prove that (adj adj A) = $|A|^{n-2} A$

(ii) Find the value of |adj adj adj A| in terms of |A|

6. For the matrix $A = \begin{bmatrix} 3 & 2 \\ 1 & 1 \end{bmatrix}$ find a & b so that $A^2 + aA + bI = 0$. hence find A^{-1}.

7. If $A^{-1} = \begin{bmatrix} 3 & -1 & 1 \\ -15 & 6 & -5 \\ 5 & -2 & 2 \end{bmatrix}$ & $B = \begin{bmatrix} 1 & 2 & -2 \\ -1 & 3 & 0 \\ 0 & -2 & 1 \end{bmatrix}$, find $(AB)^{-1}$

8. If $A = \begin{bmatrix} 2 & -1 \\ 3 & 4 \end{bmatrix}$, $B = \begin{bmatrix} 5 & 2 \\ 7 & 4 \end{bmatrix}$, $C = \begin{bmatrix} 2 & 5 \\ 3 & 8 \end{bmatrix}$ and AB – CD=0 find D.

9. If A and B are two square matrices such that AB = A & BA = B, prove that A & B are idempotent.

10. Show that $\begin{bmatrix} ab & b^2 \\ -a^2 & -ab \end{bmatrix}$ is a nilpotent matrix.

11. Find $\left\{\frac{1}{2}(A - A' + I)\right\}^{-1}$ for $A = \begin{bmatrix} -2 & 3 & 4 \\ 5 & -4 & -3 \\ 7 & 2 & 9 \end{bmatrix}$ using elementary transformation.

12. Given $A = \begin{bmatrix} 2 & 0 & -\alpha \\ 5 & \alpha & 0 \\ 0 & \alpha & 3 \end{bmatrix}$ For what values of α does A^{-1} exists. Find A^{-1} & prove that $A^{-1} = A^2 - 6A + 11I$ when $\alpha = 1$.

13. Gaurav purchases 3 pens, 2 bags and 1 instrument box and pays Rs. 41. From the same shop Dheeraj purchases 2 pens, 1 bag and 2 instrument boxes and pays Rs. 29, while Ankur purchases 2 pens, 2 bags and 2 instrument boxes and pays Rs. 44. Translate the problem into a system of equations. Solve the system of equations by matrix method and hence find the cost of 1 pen, 1 bag and 1 instrument box.

14. Solve the following system of linear equations by using the principle of matrix.

(i) $2x - y + 3z = 8$
$-x + 2y + z = 4$
$3x + y - 4z = 0$

(ii) $x + y + z = 8$
$2x+5y+7z = 52$
$2x + y - z = 0$

15. Compute A^{-1}, if $A = \begin{bmatrix} 3 & -2 & 3 \\ 2 & 1 & -1 \\ 4 & -3 & 2 \end{bmatrix}$. Hence solve the system of equations $\begin{bmatrix} 3 & 0 & 3 \\ 2 & 1 & 0 \\ 4 & 0 & 2 \end{bmatrix}\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 8 \\ 1 \\ 4 \end{bmatrix} + \begin{bmatrix} 2y \\ z \\ 3y \end{bmatrix}$.

16. If $A = \begin{bmatrix} 1 & \tan x \\ -\tan x & 1 \end{bmatrix}$, show that $A^T A^{-1} = \begin{bmatrix} \cos 2x & -\sin 2x \\ \sin 2x & \cos 2x \end{bmatrix}$.

17. Find the values of x, y, z if the matrix $A = \begin{bmatrix} 0 & 2y & z \\ x & y & -z \\ x & -y & z \end{bmatrix}$ obeys the law $A^T A = I$.

18. Compute A^{-1} for the following matrix $A = \begin{bmatrix} -1 & 2 & 5 \\ 2 & -3 & 1 \\ -1 & 1 & 1 \end{bmatrix}$. Hence solve the system of equations $-x + 2y + 5z = 2$; $2x - 3y + z = 15$ & $-x + y + z = -3$

19. If $A = \begin{bmatrix} 1 & -2 & 0 \\ 2 & 1 & 3 \\ 0 & -2 & 1 \end{bmatrix}$, find A^{-1}. Using A^{-1}, solve the system of linear equations, $x - 2y = 10$, $2x + y + 3z = 8$, $-2y + z = 7$.

20. If $A = \begin{bmatrix} 1 & -1 \\ 2 & -1 \end{bmatrix}$, $B = \begin{bmatrix} a & 1 \\ b & -1 \end{bmatrix}$ and $(A + B)^2 = A^2 + B^2$, find a and b.

21. By using the principle of matrix, show that the following system of equations has infinite solution:
$5x + 3y + 7z = 4$; $3x + 26y + 2z = 9$; $7x + 2y + 10z = 5$.

22. If the determinant $\begin{vmatrix} \sin\theta & 1 & 0 \\ 1 & \cos\phi & -\cos\theta \\ \sin\phi & 0 & 1 \end{vmatrix}$ is a symmetric determinant then find minimum and maximum value of determinant.

23. If $\begin{vmatrix} e^x & \sin x \\ \cos x & \ell n(1+x) \end{vmatrix} = A + Bx + Cx^2 + \ldots\ldots.$, then find the value of A and B.

24. Show that $\Delta = \begin{vmatrix} b^2+c^2 & ab & ac \\ ab & c^2+a^2 & bc \\ ca & cb & a^2+b^2 \end{vmatrix} = 4a^2b^2c^2$

25. Show that $\begin{vmatrix} a_1l_1+b_1m_1 & a_1l_2+b_1m_2 & a_1l_3+b_1m_3 \\ a_2l_1+b_2m_1 & a_2l_2+b_2m_2 & a_2l_3+b_2m_3 \\ a_3l_1+b_3m_1 & a_3l_2+b_3m_2 & a_3l_3+b_3m_3 \end{vmatrix} = 0$

26. Investigate for what values of λ, μ the simultaneous equations $x + y + z = 6$; $x + 2y + 3z = 10$ &
$x + 2y + \lambda z = \mu$ have;
(a) A unique solution
(b) An infinite number of solutions
(c) No solution.